Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS (PRINCIPAL) **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2016**

2 hours

* 5 3 0 6 4 0 4 2 0 5 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **3** printed pages and **1** blank page.

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1 Find the equation of the line perpendicular to the line $y = 5x + 6$ which passes through the point $(1, 11)$. Give your answer in the form $y = mx + c$. [3]

2 Without using a calculator, simplify the following, giving each answer in the form $a\sqrt{5}$ where a is an integer. Show all your working.

(i) $4\sqrt{10} \times \sqrt{2}$ [2]

(ii) $\sqrt{500} + \sqrt{125}$ [2]

3 Solve $3x^2 + 11x - 20 > 0$. [4]

4 A sequence $u_1, u_2, u_3, \ldots$, is defined by $u_n = 3n + 5$.

(i) State the values of u_1, u_2 and u_3. [1]

(ii) Find the value of n such that $u_n = 254$. [2]

(iii) Evaluate $\sum_{n=1}^{500} u_n$. [3]

5 The circle with equation $x^2 + y^2 - 6x - k = 0$ has radius 5. Find the coordinates of the centre of the circle and the value of k. [4]

6 **(i)** Find the coordinates of the stationary points of the curve with equation

$$y = 3x^4 - 20x^3 + 36x^2$$

and determine their nature. [6]

(ii) Sketch the graph of $y = 3x^4 - 20x^3 + 36x^2$ and hence state the set of values of k for which the equation $3x^4 - 20x^3 + 36x^2 = k$ has exactly four distinct real roots. [3]

7 The functions f and g are defined for all real numbers by

$$\mathrm{f}(x) = x^2 + 2 \quad \text{and} \quad \mathrm{g}(x) = 4x + 3.$$

(i) State the range of each of the functions f and g. [2]

(ii) Find the values of x for which $\mathrm{fg}(x) = \mathrm{gf}(x)$. [4]

(iii) The function h, given by $\mathrm{h}(x) = x^2 + 2$, has the same range as f but is such that $\mathrm{h}^{-1}(x)$ exists. State a possible domain for h and find an expression for $\mathrm{h}^{-1}(x)$. [2]

8 **(a)** Evaluate exactly $\int_0^1 x\mathrm{e}^{-x}\,\mathrm{d}x$. [4]

(b) Find $\int \frac{x-1}{x+1}\,\mathrm{d}x$. [4]

9 Determine whether the lines whose equations are

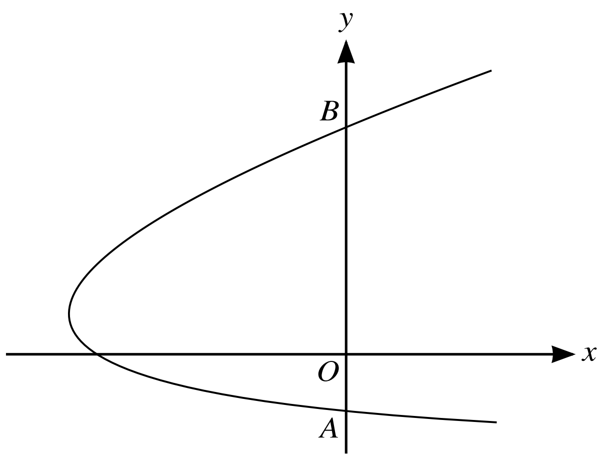
$$\mathbf{r} = (4 + 2\mu)\mathbf{i} + (7 + 3\mu)\mathbf{j} + (3 + 7\mu)\mathbf{k} \quad \text{and} \quad \mathbf{r} = (35 - 5\lambda)\mathbf{i} + (6 + 2\lambda)\mathbf{j} + (14 + 3\lambda)\mathbf{k}$$

intersect, are parallel or are skew. [6]

10 The diagram shows the curve with equation

$$x = (y - 4)\ln(2y + 3).$$

The curve crosses the y-axis at A and B.



(i) Find an expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. [3]

(ii) Find the exact gradient of the curve at each of the points A and B. [6]

11 **(i)** Prove that

$$\sin^2(\theta + \tfrac{1}{3}\pi) + \tfrac{1}{2}\sin^2\theta - \tfrac{3}{4} = \tfrac{1}{4}\sqrt{3}\sin 2\theta. \quad [4]$$

(ii) Hence solve the equation

$$2\sin^2(\theta + \tfrac{1}{3}\pi) + \sin^2\theta = 1 \quad \text{for } -\pi \leqslant \theta \leqslant \pi. \quad [5]$$

12 A patch of disease on a leaf is being chemically treated. At time t days after treatment starts, its length is x cm and the rate of decrease of its length is observed to be inversely proportional to the square root of its length. At time $t = 3$, $x = 4$ and, at this instant, the length is decreasing at 0.05 cm per day.

Write down and solve a differential equation to model this situation. Hence find the time it takes for the length to decrease to 0.01 cm. [10]

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